Quiz 3: Nov 17

<u>Last time</u>: • Solutions to the DE

$$\frac{dP}{dt} = \underline{k}P \rightsquigarrow P(t) = Ce^{kt}$$

$k>0 \rightsquigarrow$ exponential growth

$k<0 \rightsquigarrow$ exponential decay

• Easy to check sols

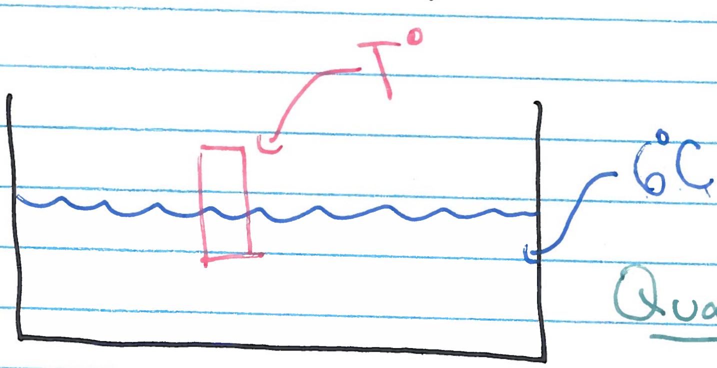
Hard to find

<u>Today</u>: • Qualitative Analysis

⇝ when you can't explicitly solve (#2)

• How to write down a DE given a word problem (#1)

<u>#2:</u>



$$\frac{dT}{dt} = -5T + 30$$

<u>Qualitative analysis</u>

• Can be done ~~on~~ DE's of the form

$$\frac{dT}{dt} = \boxed{f(T)}$$

← depends <u>only</u> on T.

ex. of non-autonomous:

$$\frac{dT}{dt} = T + t^2$$

<u>Step1</u>: Find the <u>steady states</u>. Find the solutions of the form $T(t) = C$ ← constant.

Newton's Law of Cooling

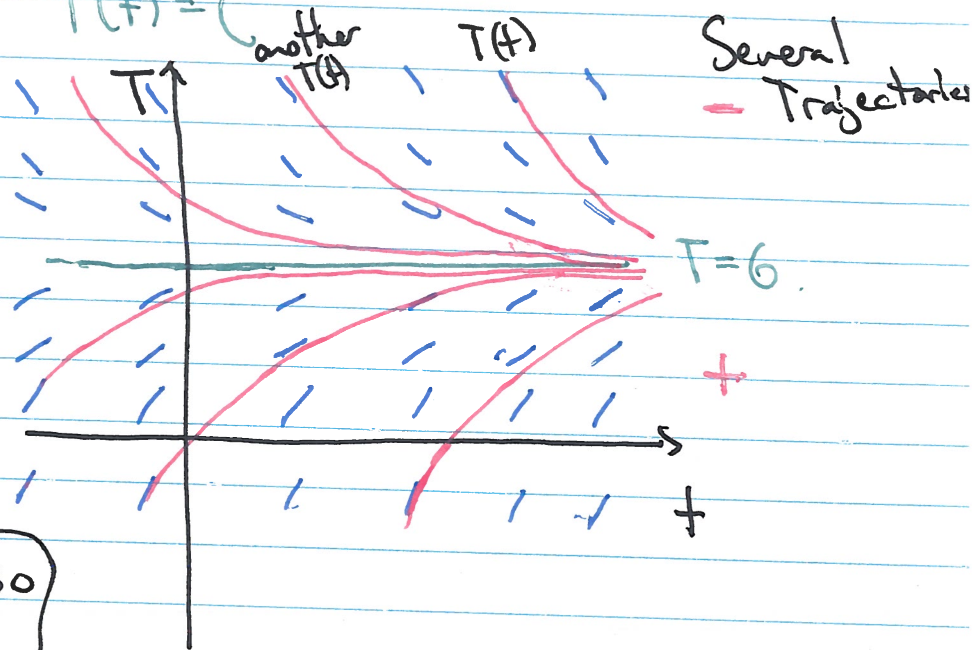
Step 2: Orient each region as ↗ or ↘

$$\frac{dT}{dt} = -5T + 30$$

$T(t) = C$

$$0 = -5C + 30$$

$$C = 6.$$

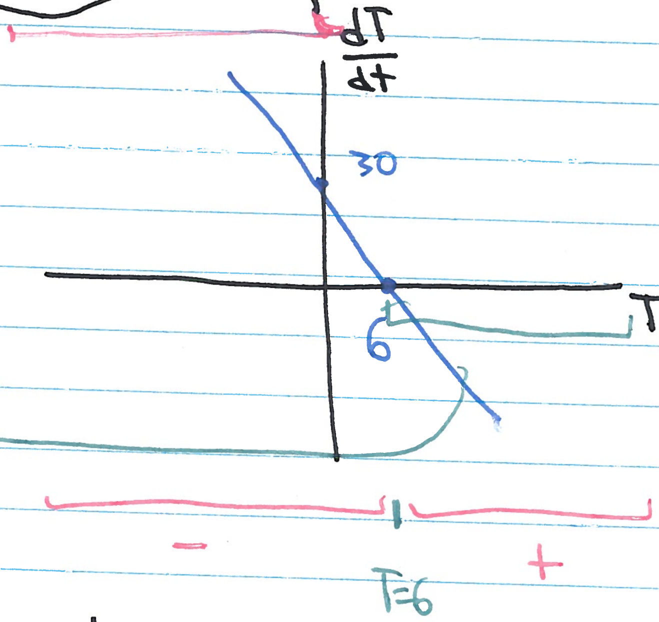


• For $T < 6$,

$-5T + 30 = -(\text{something} + 30)$
$<30)$
>0

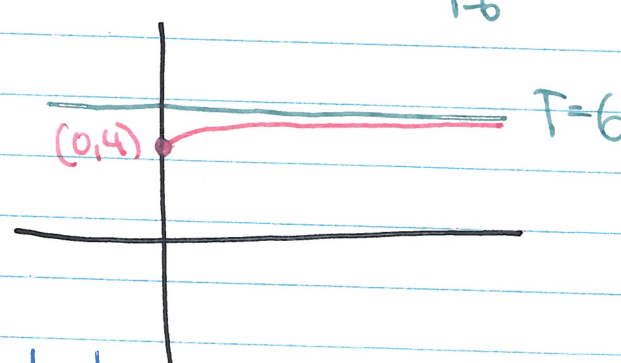
• For $T > 6$,

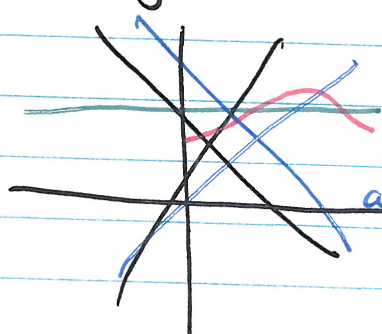
$-5T + 30 < 0$



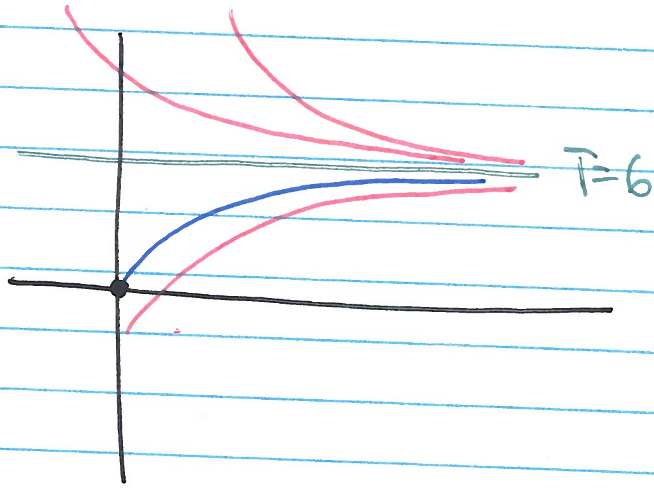
Step 3: Sketch in trajectories

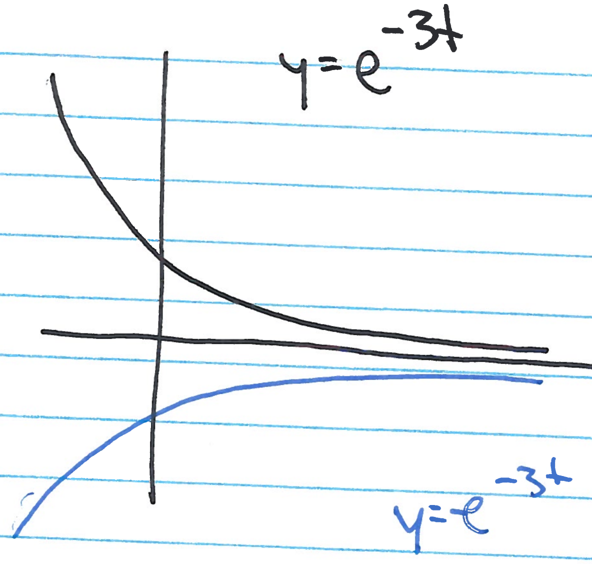
Ex. given $T(0) = 4°$





Cannot cross a steady trajectory!





Guess: Maybe the solutions look like

$$T(t) = 6 + Ce^{-kt} \quad \text{for some } C, k?$$

Try plugging in!

$$\frac{dT}{dt} = -5T + 30$$

$$-kCe^{-kt} = \cancel{-30} - 5Ce^{-kt} + \cancel{30}$$

$$-kCe^{-kt} = -5Ce^{-kt} \rightsquigarrow k=5, \ C \text{ any constant.}$$

$$\rightsquigarrow \boxed{T(t) = 6 + Ce^{-5t}}$$

Same technique for $\frac{dT}{dt} = aT + b$ Linear ODE Ordinary Diff. Eq

Initial Condition: $T(0) = 0$.

$$0 = 6 + C\overbrace{e^{-5\cdot 0}}^{=1}$$

$$0 = 6 + C \implies C = -6.$$

$$\implies \boxed{T(t) = 6 - 6e^{-5t}}$$